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FUZZY SETS AND MULTI-DESCRIPTION PROPERTY

Chee Kau Lim and Chee Seng Chan
limck@siswa.um.edu.my cs.chan@um.edu.my
Faculty of Computer Science and Information Technology, University of Malaya, 50603 Kuala Lumpur, Malaysia

PROBLEM STATEMENTS

One used to define membership functions as simple scalar values in [0,1], but we noticed that this does not apply to multi-description sets.

Membership functions of multi-description sets posses multi-description property, and should be determined by multiple partially independent descriptions, instead of one. For example, Asian Food is a multi-description set.

In multi-description sets, each description carries certain weight, d_i to constitute the membership function:

$$\sum_{i=1}^{N} d_i = 1$$

Membership function of an element is determined by :

$$\mu_A(x) = \sum_{i=1}^{N} d_i c_i \in [0,1]$$

where c_i is the strength of the element x in A in showing description d_i.

THE IMPACT

Due to the membership functions are "partitioned" by several descriptions, fundamental operations such as min t-norm, max t-conorm and subsethood measure [1]:

$$\pi(B \subseteq A) = 1 - \frac{\sum_{x \in X} \max(0, \mu_B(x) - \mu_A(x))}{\sum_{x \in X} \mu_B(x)}$$

do not provide expected results.

For example, fuzzy sets of Asian Food, A and My Favourite Food, B are in the universe Food, X. For an arbitrary food x, if $\mu_A(x) = 0.8$ and $\mu_B(x) = 0.6$, without considering multi-description property, we can easily find $A \cup B$=0.8, $A \cap B$=0.6 and $\Pi(B \subseteq A)$=1.0.

However, Food is a multi-description set. Assume that a food can be identified by 3 descriptions with corresponding weight: d_1=0.3, d_2=0.3 and d_3=0.4. We can present the union, intersection and subsethood measure of x under two different scenarios, as shown in Table I and II, respectively.

Table 1: Best Matching Scenario

Set	$d_1\,c_1$	$d_2\,c_2$	$d_3\,c_3$	membership function
A	0.3	0.3	0.2	0.8
B	0.3	0.3	0	0.6
$A \cap B$	0.3	0.3	0.2	0.6
$A \cup B$	0.3	0.3	0.2	0.8

In the scenario of Table I, the $\mu_A(x)$ and $\mu_B(x)$ is given by the matrix of c_i [1,1,0.5] and [1,1,0] respectively. Although the measurements of fundamental operations proposed earlier are correct, but this scenario only represents an extreme case.

Table 2: Scattered Membership Function Scenario

Set	$d_1\,c_1$	$d_2\,c_2$	$d_3\,c_3$	membership function
A	0.3	0.1	0.4	0.8
B	0.2	0.2	0.2	0.6
$A \cap B$	0.2	0.1	0.2	0.5
$A \cup B$	0.3	0.2	0.4	0.9

From Table 2, it is clear that if $\mu_A(x)$ and $\mu_B(x)$ is given by the matrix of c_i [1,0.33,1] and [0.67,0.67,0.5] respectively, the measurements of fundamental operations proposed earlier are not correct.

Variation of c_i may cause the measurements of fundamental operations change accordingly. Hence, the classical min t-norm, max t-conorm and subsethood measure are not compatible to the multi-description property.

PROPOSED SOLUTION

Modelling a multi-description set with multiple fuzzy sets or Type-2 fuzzy sets [2] is not a good solution for the problem.

We propose to solve the problem with Checklist Paradigm [3].

For each operation, we use Checklist Paradigm to find upper and lower bounds of the measurement, as well as an expected value. Instead of the extreme cases represented by the upper and lower bounds, the expected value is used as the more reliable measurements.

SOLUTION FOR TYPE-1 FUZZY SETS

Consider a list of descriptions d_i, $i \in \{1, 2, \ldots, \kappa\}$ for both sets A and B. By studying the strength of x in showing each description for A and B, we can find $\mu_A(x)=a$ and $\mu_B(x)=b$. With a and b, Table III is formed.

Table III : The matrix of relation between A and B

	A	A'	total
B	$s_{00} = \mu_{A \cap B}(x)$	$s_{10} = \mu_{A' \cap B}(x)$	b
B'	$s_{01} = \mu_{A \cap B'}(x)$	$s_{11} = \mu_{A' \cap B'}(x)$	1-b
total	a	1-a	1

We also can rewrite Table III in term of a, b and θ, where $\theta = \mu_{A \cap B}(x)$ (Table IV).

Table IV : The relation of A and B in term of θ

	A	A'	total
B	θ	$b - \theta$	b
B'	$a - \theta$	$1 - a - b + \theta$	1-b
total	a	1-a	1

In term of cells:

$$A \cap B(x) = S_{00}$$

$$A \cup B(x) = S_{00} + S_{01} + S_{10}$$

For subsethood measures, we adopt the definition based on fuzzy implication operator, → [4]:

$$\pi(A \subseteq B) = \bigwedge_{x \in X} \left(\mu_A(x) \to \mu_B(x)\right)$$

Therefore,

$$\pi(A \subseteq B) = S_{00} + S_{10} + S_{10}$$

Consider when the θ varies, if a and b are fixed, we can find the bounds and expected values for each operation, summarized in Table V. Expected values are obtained when the values in the cells distributed evenly. Although the expected values may not reflect the exact measures, but they represent the measures with highest probability.

EXTENSION TO TYPE-2 FUZZY SETS

Type-2 fuzzy sets (T2FS) cannot be avoided from the influence of multi-description property. Therefore, we extend the result of Type-1 fuzzy sets (T1FS) to provide T2FS subsethood measures. This T2FS subsethood measures is tolerable to multi-description property.

We deduce that subsethood measure for T2FS should be intervals or T1FS, instead of single point values.

Representative Method: Find a compatible embedded T1FS [5] to represent an interval T2FS and find subsethood measurement based on this T1FS. This compatible T1FS can be found by joining up all the center-of-areas of vertical slices (Fig 1).

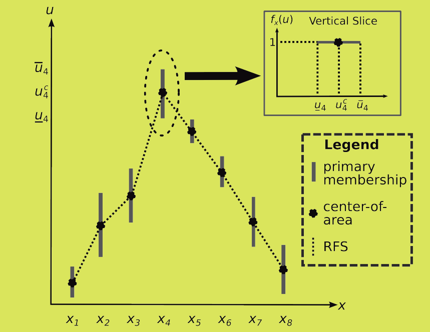

Fig 1 : Once the center-of-area of all the elements are obtained, union of these points form a representative fuzzy set.

Instead of directly working on interval T2FS, representative fuzzy sets provide opportunity to perform simple and fast computation for approximate results. Techniques for T1FS can be used directly to obtain subsethood measures for interval T2FS. However, in order to make the subsethood measures more reliable, intervals are proposed instead of point values. These intervals are correponding to lower and uppper bounds of subsethood measures, i.e:

$$\pi(\tilde{A} \subseteq \tilde{B}) = [sub_l, sub_u]$$

where:

$$sub_l = \frac{1}{N} \sum_{x \in X} max(v^c, 1 - u^c)$$

$$sub_u = \frac{1}{N} \sum_{x \in X} min(1, 1 - u^c + v^c)$$

For general T2FS, an embedded T2FS is formed wiht the representative method. The results of subsethood measures are fuzzy sets with triangular membership functions instead of intervals. The height of these fuzzy sets are δ at sub_m (Fig 2), where:

$$\delta = \min_{x \in X} \left(f_x(u^c), g_x(v^c)\right)$$

$$sub_m = \frac{1}{N} \sum_{x \in X} \left(1 - u^c + u^c v^c\right)$$

Fig 2 : T1FS for subsethood measurements of general T2FS.

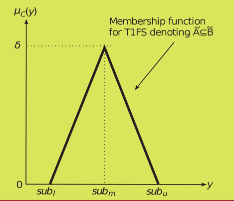

Table V. Summary of bounds and expected values of intersection, union and subsethood measures.

	Lower bound	Expected value	Upper bound
Intersection	max (0, a+b-1)	ab	min (a,b)
Union	max (a,b)	a+b-ab	min (1, 1-a+b)
Subsethood	$\bigwedge_{x \in X}$ max (b, 1-a)	$\bigwedge_{x \in X}$ (1-a+ ab)	$\bigwedge_{x \in X}$ min (1, 1-a+b)

CONCLUSIONS

In this research, we highlighted an unattended property of fuzzy sets, namely the multi-description property. For fuzzy sets that exhibit this property, min t-norm, max t-conorm and Kosko subsethood measure do not give complete results. With Checklist Paradigm, we identify a set of equations that provides more reasonable results.

We also extend the solution of subsethood measures to Type-2 fuzzy sets with Representation Method. To make this measure more reliable, the results are given as intervals or type-1 fuzzy sets, instead of point values.

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